



GRADE 4 MATH: CHOCOLATE BAR FRACTIONS

UNIT OVERVIEW

In this unit students apply their knowledge of equivalence, ordering, and addition with fractions and the meaning of multiplication to extend their understanding of multiplication with fractions by whole numbers. This unit will take approximately 4-5 weeks.

TASK DETAILS

Task Name: Chocolate Bar Fractions

Grade: 4

Subject: Mathematics

Depth of Knowledge: 2

Task Description: This task asks students to multiply a fraction by a whole number and reason about the meaning of a solution that includes a fractional chocolate bar. Students are also asked to construct and critique arguments by reasoning about the products of whole numbers and fractions.

Standards:

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

- 4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
- Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (\frac{1}{4})$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (\frac{1}{4})$.
 - Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (\frac{2}{5})$ as $6 \times (\frac{1}{5})$, recognizing this product as $\frac{6}{5}$. (In general, $n \times (\frac{a}{b}) = (n \times a)/b$.)
 - Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Standards for Mathematical Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.



TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement Common Core–aligned tasks that are embedded in a unit of instruction. We have learned through our pilot work that focusing instruction on units anchored in rigorous Common Core–aligned assessments drives significant shifts in curriculum and pedagogy. Callout boxes and Universal Design for Learning (UDL) support are included to provide multiple entry points for diverse learners.

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GRADE 4 MATH: CHOCOLATE BAR FRACTIONS PERFORMANCE TASK

CHOCOLATE BAR FRACTIONS

Part 1

John is giving out chocolate to his friends. If he wants to give each friend $\frac{2}{3}$ of a chocolate bar and he has 13 friends, how many chocolate bars will he need to buy?
Use words, a model, or an equation to justify your answer.

Part 2

William buys 4 chocolate bars and each bar weighs $\frac{1}{4}$ pound. Mary buys 2 chocolate bars and each one weighs $\frac{1}{2}$ pound. William claims that the chocolate weighs the same amount. Mary disagrees. Who is correct? *Use a model and words to justify your answer.*



GRADE 4 MATH: CHOCOLATE BAR FRACTIONS

RUBRIC

The rubric section contains performance level descriptions for the Chocolate Bar Fractions task. Performance level descriptions help teachers consider the overall qualities of work for the task by providing information about the expected level of performance for students.

Performance Level 4

Student correctly multiplies 13 by $\frac{2}{3}$ to arrive at a correct product of $\frac{26}{3}$. Student correctly interprets $\frac{26}{3}$ as 8 and $\frac{2}{3}$ and recognizes that John requires 9 candy bars to share among his friends. Student uses an appropriate strategy such as a number line, visual fraction model, or algorithm to multiply the fraction by the whole number and explains that John needs 9 candy bars because he cannot buy 8 and $\frac{2}{3}$ candy bars.

Student correctly identifies that $\frac{4}{4}$ and $\frac{2}{2}$ are equivalent and states that William is correct. Student includes a model and justifies answer using reasoning such as:

- 4 pieces that are each $\frac{1}{4}$ make one whole and 2 pieces that are each $\frac{1}{2}$ make one whole because in both cases we have all of the pieces or one whole, so $\frac{4}{4} = \frac{2}{2}$
- $\frac{2}{2} \times \frac{2}{2}$ equals $\frac{4}{4}$ or $\frac{4}{4} \times (\frac{1}{2})/(\frac{1}{2})$ equals $\frac{2}{2}$
- If you have 2 pieces that are each a half and you cut the two halves into two equal pieces you get fourths. Since both halves belonged to you because you had $\frac{2}{2}$, now you have $\frac{4}{4}$, or the same amount.

Student makes sense of the problem and applies knowledge of fractions to provide an accurate solution. Student uses clear language to communicate written responses. In written explanations, student refers to labels, quantities, and units precisely such as referring correctly to units as either chocolate bars in part 1 or pounds of chocolate in part 2. Models including number lines or area models are appropriate, clearly reflecting the problem situation. The student supports his/her responses with logical and appropriate reasoning.

Performance Level 3

Student correctly multiplies 13 by $\frac{2}{3}$, with a correct product of $\frac{26}{3}$. Student may not interpret $\frac{26}{3}$ as 8 and $\frac{2}{3}$ or may not recognize that John requires 9 candy bars to share among his friends. Student uses an appropriate strategy such as a number line, visual fraction model, or algorithm to multiply the fraction by the whole number.

Student correctly identifies that $\frac{4}{4}$ and $\frac{2}{2}$ are equivalent. Student explains answer in words or uses a diagram such as a number line, area model, or an equation. Reasoning is generally correct, though explanation may be limited.

Student makes sense of the problem and applies knowledge of fractions and operations to provide an accurate solution. Student uses clear language to communicate written responses. In written explanations, student refers to labels, quantities, and units. Models are appropriate, reflecting the problem situation. The student supports his/her responses with reasoning.

Performance Level 2

Student attempts to multiply 13 by $\frac{2}{3}$, with an incorrect product or a number line, visual fraction model, or algorithm that indicates a conceptual error. Student may add $\frac{2}{3}$ repeatedly or try to partition 13 into 3 equal groups, with limited success. Student is unable to identify either the number of candy bars that John intends to distribute (8 and $\frac{2}{3}$) or the number he needs to buy (9).

Student attempts to explain why fractions are/are not equivalent using an appropriate strategy, but may incorrectly multiply $\frac{1}{2}$ by 2 or $\frac{1}{4}$ by 4.

Student communicates an incomplete argument with unclear reference to quantities, units, and labels. The student may generally describe fractional equivalence. Student may apply an algorithm inappropriately or with limited evidence of understanding.

Performance Level 1

Student attempts to solve the problem, but work demonstrates major conceptual flaws. Student provides very limited evidence of understanding the operations required to solve the problem such as being unable to generate the correct weight of the chocolate or demonstrate fractional equivalence. Work may include an answer such as “William” or “Mary” with no work or justification or an incorrect justification that indicates a major conceptual error.



GRADE 4 MATH: CHOCOLATE BAR FRACTIONS

ANNOTATED STUDENT WORK

This section contains annotated student work at a range of performance levels. The student work shows examples of understandings and misunderstandings of the task.

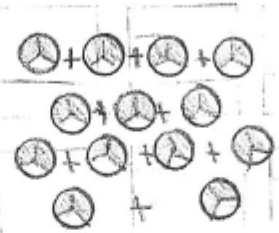
CHOCOLATE BAR FRACTIONS TASK: PERFORMANCE LEVEL 4

CHOCOLATE BAR FRACTIONS

Part 1
John is giving out chocolate to his friends. If he wants to give each friend $\frac{2}{3}$ of a chocolate bar and he has 13 friends, how many chocolate bars will he need to buy?
Use words, a model, or an equation to justify your answer.

Answer: 9 chocolate bars

Equation: $\frac{2}{3} \times 13 = \frac{2}{3} \times 13 \frac{1}{1} = \frac{26}{3} = 8 \frac{2}{3}$

Model: 

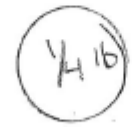
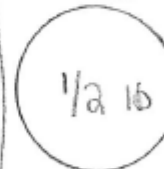

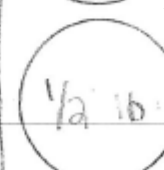


Words: Since there were 13 friends (not including John) $\frac{2}{3} \times 13 = 8 \frac{2}{3}$.

Part 2
William buys 4 chocolate bars and each bar weighs $\frac{1}{4}$ pound. Mary buys 2 chocolate bars and each one weighs $\frac{1}{2}$ pound. William claims that the chocolate weighs the same amount. Mary disagrees. Who is correct? *Use a model and words to justify your answer.*

Answer: William

Equation: $\frac{1}{4} \times 4 = 1$; $\frac{1}{2} \times 2 = 1$
 $\frac{1}{4} \times 4 = \frac{1}{2} \times 2$

Model:

William	Mary
	
	
	
	
Totals → 1 lb	1 lb

Words: $\frac{1}{4} \times 4 = 1$. 4 $\frac{1}{4}$'s would be the same as $\frac{1}{2} \times 2$. 4 split parts is 1. 2 split parts is 1.

4.NF.4 Student uses a visual fraction model, an equation, and words to represent $\frac{26}{3}$ as 8 and $\frac{2}{3}$.
MP.1, MP.4, MP.6, MP.7

4.NF.2 Student compares two fractions with unlike numerators and denominators by relating them to the whole as a visual fraction model, and is able to use an equation and words to justify the answer to represent $\frac{26}{3}$ as 8 and $\frac{2}{3}$.
MP.1, MP.4, MP.6, MP.7

Performance Level 4

The student's response meets the demands of the entire task and provides multiple models to represent the problem. In part 1, the student represents the multiplication of $\frac{2}{3}$ and 13 as the repeated addition of $\frac{2}{3}$. The circles drawn are $\frac{2}{3}$ shaded, and the student correctly identifies the solution to this repeated addition as $\frac{26}{3}$ or 8 and $\frac{2}{3}$. The student also uses a fraction multiplication algorithm to write an equation to represent the multiplication. The student correctly determines that John must buy

9 chocolate bars, even though the product obtained earlier is $8\frac{2}{3}$. No additional explanation is provided to contextualize or explain the conclusion that 9 chocolate bars are required.

The student in part 2 identifies that William is correct and represents the problem using a diagram. The two diagrams do not represent halves and fourths in a way that allows for comparison among the parts or the whole. While the half is appropriately larger than the fourth, it is not immediately evident that the area/size of the fourth is half that of the half. The student provides a description in words that recognizes the relationship between the whole and the number of parts into which the whole is split.

CHOCOLATE BAR FRACTIONS TASK: PERFORMANCE LEVEL 3

CHOCOLATE BAR FRACTIONS

Part 1
John is giving out chocolate to his friends. If he wants to give each friend $\frac{2}{3}$ of a chocolate bar and he has 13 friends, how many chocolate bars will he need to buy? Use words, a model, or an equation to justify your answer.

Part 2
William buys 4 chocolate bars and each bar weighs $\frac{1}{4}$ pound. Mary buys 2 chocolate bars and each one weighs $\frac{1}{2}$ pound. William claims that the chocolate weighs the same amount. Mary disagrees. Who is correct? Use a model and words to justify your answer.

William is correct because $4 \times \frac{1}{4} = 1$ just like $2 \times \frac{1}{2} = 1$. What I did to get the answer was I made a model that was cut in half then another model cut in fourth.

MP.4, MP.7
Student is able in part 1 and part 2 to make use of structure by using visual fraction models and symbols to arrive at a correct answer.

MP.3 Student is able to explain how the correct answers were obtained; however, the student does not justify the answers by using an equation to show the relationship between multiplication of a fraction by a whole number.

Performance Level 3

This student's response meets the demands of the task. The student uses a visual model of thirds (circles divided into three parts) and labels each third with the number of each friend with whom the chocolate bar was shared. One section remains unlabeled. 8 and $\frac{2}{3}$ sections are labeled, and the student indicates that John will need to buy "9 bars." While the student explains the process used to draw the diagram, s/he does not identify the number of labeled segments (or the fact that $\frac{26}{3}$ are required for John's friends). When referring to the model, the student does not indicate what represents the whole (e.g., "making a model with circles that represent candy bars"). The student also doesn't justify the fact that John needs to buy 9 candy bars as opposed to the $\frac{26}{3}$ (or 8 and $\frac{2}{3}$) that represents the product of $\frac{2}{3}$ and 13.

In part 2, the student uses a diagram to represent the addition of two halves to get one pound and four fourths to get a pound. The diagram uses congruent rectangles to represent the whole in both situations. The student identifies that William is correct and communicates an equivalence between the multiplicative relationships and 1. However, the equations written in the explanation of part 2 don't properly identify multiplication as the operation on 4 and $\frac{1}{4}$ and 2 and $\frac{1}{2}$. The student effectively uses models to represent the problem, but doesn't sufficiently communicate his/her understanding of the model and its relationship to the original situation.

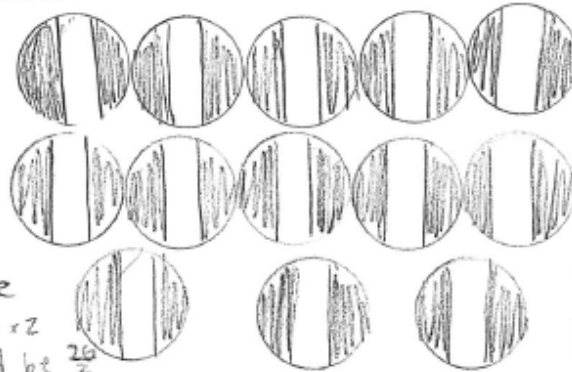
CHOCOLATE BAR FRACTIONS TASK: PERFORMANCE LEVEL 2

CHOCOLATE BAR FRACTIONS

Part 1

John is giving out chocolate to his friends. If he wants to give each friend $\frac{2}{3}$ of a chocolate bar and he has 13 friends, how many chocolate bars will he need to buy? Use words, a model, or an equation to justify your answer.

$$\frac{2}{3} \times 13 = \frac{26}{3}$$



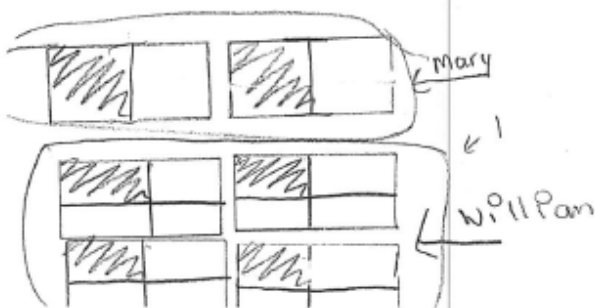
26 chocolate bars because 13×2 is 26 and it could be $\frac{26}{3}$

Part 2

William buys 4 chocolate bars and each bar weighs $\frac{1}{4}$ pound. Mary buys 2 chocolate bars and each one weighs $\frac{1}{2}$ pound. William claims that the chocolate weighs the same amount. Mary disagrees. Who is correct? Use a model and words to justify your answer.

$$\frac{1}{4} \times 4 = \frac{4}{4} = 1 \text{ whole}$$

$$\frac{1}{2} \times 2 = \frac{2}{2} = 1 \text{ whole}$$



William is correct because they both have 1 whole chocolate bar. When you multiply $\frac{1}{2}$ by 2 you get $\frac{2}{2}$. And when you multiply $\frac{1}{4}$ by 4 you get $\frac{4}{4}$.

4.NF.4

Student is able to correctly use multiplication to get $\frac{26}{3}$, but lacks the understanding of how to convert an improper fraction to a mixed number.

MP.4

MP.1, MP.3, MP.4

Student is able to correctly use unit fractions to multiply a fraction by a whole number. Student does not attend to precision when referring to "1 whole chocolate bar" instead of pounds of chocolate.

Performance Level 2

This student's response meets some of the demands of the task. In part 1, the student's work reflects a conceptual misunderstanding, insofar as the pieces of each circle are not of equal size. While the student correctly multiplies $\frac{2}{3}$ by 13 to get $\frac{26}{3}$, the student is unable to contextualize the response and incorrectly indicates that John needs to buy 26 chocolate bars.

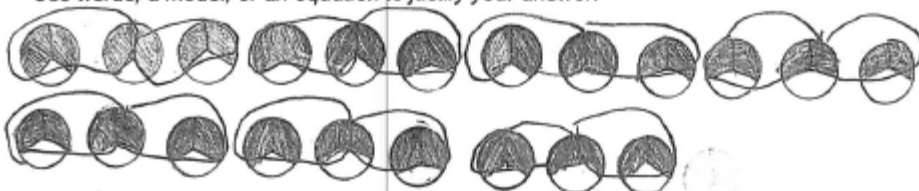
In part 2, the student correctly multiplies both $\frac{1}{4}$ by 4 and $\frac{1}{2}$ by 2 to get 1 whole. The student doesn't attend to precision when s/he indicates that they represent whole chocolate bars, rather than pounds of chocolate. While the student was able to multiply correctly, the response demonstrates that s/he is unable to contextualize arithmetic and make sense of problems.

CHOCOLATE BAR FRACTIONS TASK: PERFORMANCE LEVEL 1

CHOCOLATE BAR FRACTIONS

Part 1

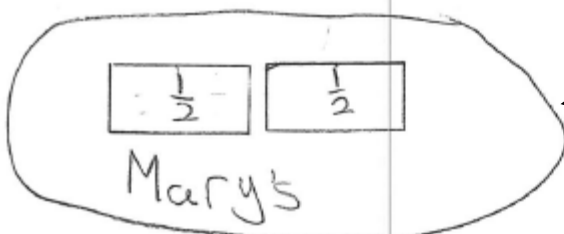
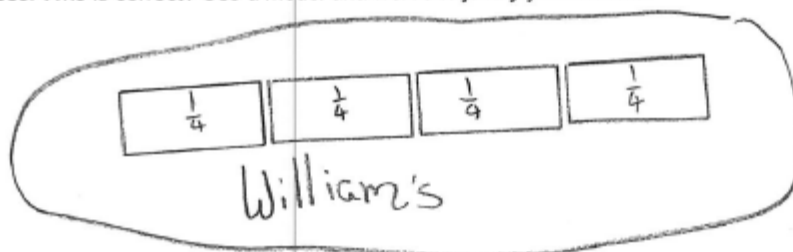
John is giving out chocolate to his friends. If he wants to give each friend $\frac{2}{3}$ of a chocolate bar and he has 13 friends, how many chocolate bars will he need to buy?
Use words, a model, or an equation to justify your answer.



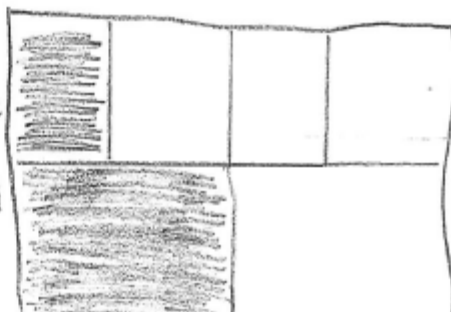
John is going to need 14 chocolate bars because he is going to give $\frac{2}{3}$ of a chocolate bar to each friend then there's going to be 1 left over.

Part 2

William buys 4 chocolate bars and each bar weighs $\frac{1}{4}$ pound. Mary buys 2 chocolate bars and each one weighs $\frac{1}{2}$ pound. William claims that the chocolate weighs the same amount. Mary disagrees. Who is correct? Use a model and words to justify your answer.



Mary is correct because if you make a model of $\frac{1}{2}$ and $\frac{1}{4}$ you will see $\frac{1}{2}$ is bigger than $\frac{1}{4}$.



Student is able to accurately represent visual models of unit fractions to multiply a fraction by a whole number (**MP.1, MP.4**). Student will need more support with extending his/her understanding of fraction models to multiplication of unit fractions with whole numbers and being able to justify the solution in words.

Performance Level 1

The student attempts to solve the problem, though there are major conceptual errors. In part 1, the student draws 21 circles, each $\frac{2}{3}$ shaded. There are 13 groups of $\frac{3}{3}$ circled ($\frac{2}{3}$ from 1 circle and $\frac{1}{3}$ from another). The student's model and written explanation demonstrate major conceptual flaws.

In part 2, the student draws two models. In one, the student draws non-proportional representations of a pound, i.e. $\frac{1}{4}$ and $\frac{1}{2}$ of a pound are represented as the same size. In this case it appears that William has more chocolate. In the second representation, the representations are proportional, but the amounts shaded do not reflect the multiplied amounts, only the original. In this case, it appears that Mary has more chocolate. The student's explanation reflects the latter representation, in which only the original fraction (representing the weight of one candy bar) is considered, not the product of the fraction and the number of candy bars each child has.



GRADE 4 MATH: CHOCOLATE BAR FRACTIONS

INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Unit Outline

INTRODUCTION: This unit outline provides an example of how to integrate performance tasks into a unit. *Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic. The length of the unit includes suggested time spent on the classroom instruction of lessons and administration of assessments. Please note that this framework does not include individual lessons.*

Grade 4 Math: Chocolate Bar Fractions

UNIT TOPIC AND LENGTH:

This 4-week unit is focused on providing students an understanding of fractions, including equivalence, ordering, and addition, which will enable them to multiply fractions by whole numbers. Students engaged in this unit should have prior knowledge of adding and subtracting fractions with like denominators. The concept of unit fractions is introduced to help students understand that a whole can be partitioned into equal parts. Unit fractions become the basic building blocks for fractions, and are reinforced through the use of number lines and number-line diagrams. Some work with equivalent fractions, including the use of number lines and fraction strips to compare fractions, prepares students in grade 3 for grade 4. In grade 3, students learned that in multiplication, objects can be grouped to show the number of objects in a set. Grade 4 students extend their understanding of multiplication by applying it to fractions. By grade 4, students have a fundamental understanding of equivalent fractions and use area models and number-line diagrams to reason about equivalence, and connect fractions with addition and multiplication. By grade 5, students connect fractions with division by using the relationship between division and multiplication to solve simple fraction division problems. They reason and make sense of fractional quantities when solving word problems and estimating answers mentally.

COMMON CORE LEARNING STANDARDS:

- **4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.
- **4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
 - a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times (\frac{1}{4})$, recording the conclusion by the equation $\frac{5}{4} = 5 \times (\frac{1}{4})$.
 - b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (\frac{2}{5})$ as $6 \times (\frac{1}{5})$, recognizing this product as $\frac{6}{5}$. (In general, $n \times (\frac{a}{b}) = (n \times a)/b$.)

<p>c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</p> <ul style="list-style-type: none"> ➤ MP.1 Make sense of problems and persevere in solving them. ➤ MP.3 Construct viable arguments and critique the reasoning of others. ➤ MP.4 Model with mathematics. ➤ MP.6 Attend to precision. ➤ MP.7 Look for and make use of structure. 	
<p>BIG IDEAS/ENDURING UNDERSTANDINGS:</p> <ul style="list-style-type: none"> ➤ Fractions are used to represent part-to-whole, division, and measurement relationships. ➤ Fractions have multiple, equivalent representations (including equivalent fractions, area models, number line models, etc.). ➤ Fractions can be compared using benchmarks, e.g., 0, $\frac{1}{2}$, 1, and visual models. ➤ Multiplication of a fraction by a whole number is related to the multiplication of a whole number by a whole number. ➤ Repeated addition of a fraction is related to multiplication of that fraction by a whole number. ➤ Fractions can be composed by successively adding the unit fraction with the same denominator. 	<p>ESSENTIAL QUESTIONS:</p> <ul style="list-style-type: none"> ➤ How can an understanding of fractions and the relationships among fractions be used to solve problems? ➤ What does it mean to multiply a fraction by a whole number? In what context would this happen? ➤ What does it mean for fractions to be equivalent? How does this understanding help to recognize and generate equivalent fractions? ➤ How is multiplying two whole numbers similar to and different from multiplying a whole number by a fraction?
<p>CONTENT:</p> <ul style="list-style-type: none"> ➤ The meaning of fractions as part-to-whole, division, and measurement relationships ➤ Relationships between parts and the whole when considering fractions ➤ The connection between a numerator and denominator and part-to-whole relationship ➤ Equivalent fractions ➤ Fraction addition ➤ Multiplication of fractions by whole numbers ➤ Comparing fractions ➤ Fraction number lines ➤ Benchmark fractions 	<p>SKILLS:</p> <ul style="list-style-type: none"> ➤ Compare fractions using $<$, $>$, or $=$. ➤ Use models to show fractional equivalence (including Cuisenaire rods, number lines, area models, etc.). ➤ Represent fractions on number lines. ➤ Use a number line to add fractions and multiply fractions by whole numbers. ➤ Use visual fraction models to show equivalence. ➤ Recognize equivalent fractions. ➤ Use an algorithm to generate equivalent fractions. ➤ Represent fractions using different

➤ Connections between whole number and fraction multiplication	models (including area models, grouping models, and number lines). ➤ Solve problems involving multiplication of a fraction by a whole number. ➤ Multiply a fraction by a whole number. ➤ Use a model to represent a mathematical problem.
VOCABULARY/KEY TERMS: <ul style="list-style-type: none"> ➤ whole ➤ equal part ➤ area model ➤ number line ➤ algorithm ➤ equation ➤ fraction ➤ numerator ➤ denominator 	
ASSESSMENT EVIDENCE AND ACTIVITIES: INITIAL ASSESSMENT: PENGUIN ICE CREAM COMPANY <i>The Penguin Ice Cream Company</i> task was used as an initial assessment of students' understanding of fraction concepts. This task assesses fraction concepts that have already been taught, including generating equivalent fractions and identifying fractional sets.	
FORMATIVE ASSESSMENT: A number of formative and instructional tasks are included with this unit and can be used to assess students' progress of the major goals of this unit. In particular, tasks such as <i>Comparing Fractions</i> and <i>Kelley's Pizza Shop</i> can be used to assess students' ability to compare fractions, identify equivalent fractions, and multiply fractions.	
FINAL PERFORMANCE TASK: The <i>Chocolate Bar Fractions</i> task requires students to multiply a fraction by a whole number and reason about the meaning of a solution that includes a fractional chocolate bar. Students are also required to construct and critique arguments by reasoning about the products of whole numbers and fractions. See the <i>Chocolate Bar Fractions</i> task below for additional details: Part 1 John is giving out chocolate to his friends. If he wants to give each friend $\frac{2}{3}$ of a chocolate bar and he has 13 friends, how many chocolate bars will he need to buy? <i>Use words, a model, or an equation to justify your answer.</i> Part 2 William buys 4 chocolate bars and each bar weighs $\frac{1}{4}$ pound. Mary buys 2 chocolate bars and each one weighs $\frac{1}{2}$ pound. William claims that the chocolate weighs the same amount. Mary disagrees. Who is correct? <i>Use a model and words to justify your answer.</i>	
LEARNING PLAN & ACTIVITIES:	

This unit consists of two distinct arcs of instruction:

- In Arc 1, connections are built between equivalence and concepts students previously learned as they studied fractions earlier in grade 4 and in grade 3. In particular, students deepen their understanding of fractions as numbers on the number line. Students are expected to understand multiple representations of fractions—including number lines, area models, and groups of objects—and represent operations of fractions using these models. In this first arc, students use these models to understand fractional equivalence.
- In Arc 2 of instruction, these models are extended as students work to represent the multiplication of fractions by whole numbers.

Arc 1

In Arc 1 of instruction, lessons bridge third and fourth grade standards and deepen student understanding of fractions as points on a number line, area models, and equivalence in preparation for the multiplication of fractions by whole numbers in Arc 2. (4.NF.2) Arc 1 should last approximately 2 weeks.

Lessons and activities could include:

- Discussions about the real world application of parts and wholes as well as fractions.
- Reinforcement of the concepts of “the whole” and “equal parts.”
- Exploration of Cuisenaire Rods to investigate the relationships between unit fractions and other fractions with the same denominator.
- Demonstrating an understanding that only fractions referring to the same whole can be compared.
- Relating the ruler to a number line and representing fractions greater than one with a fractional number line.
- Examining what fraction equivalency means.
- Folding paper to introduce the area model for fractions, attending to the fact that while the number and size of parts change, the whole stays the same.
- Modeling fractions with area, counters, and number lines.
- Utilizing benchmarks 0, $\frac{1}{2}$, and 1 to compare fractions—*Fishin’ For Fractions*. (http://www.nsa.gov/academia/files/collected_learning/elementary/arithmetic/fish_for_fract.pdf)
- Instructional task: *A Trip to the Candy Store*.

Additional Support Strategies (Arc 1): As students begin to work more deeply with fractions, vocabulary and precision become more important for effective communication. For students with language difficulties or for second language learners, consider posting a diagram of a fraction that serves as a reminder of what “numerator” and “denominator” refer to when speaking about fractions. In building student fluency with these terms, be sure to refer explicitly and visually to the part of the fraction being identified with these terms.

Arc 2

Arc 2 of instruction focuses on the connection between fraction models and multiplication of fractions by whole numbers. Students develop an understanding that a fraction is a multiple of the unit fraction with the same denominator (4.NF.4b). Arc 2 should last

approximately 2 weeks.

Lessons and activities could include:

- Exploring the relationship between multiplying two whole numbers and multiplying a whole number by a fraction using a number line.
- Utilizing unifix cubes to model multiplication of fractions.
- Multiple means of representation such as using a table to show their work in words, models, algorithms, and on number lines.
- Investigating fractions as points on a number line.
- Instructional task: ***Jetta's and Caitlyn's Brownies Task***.
- Formative Assessment: ***Kelley's pizza shop Task***.

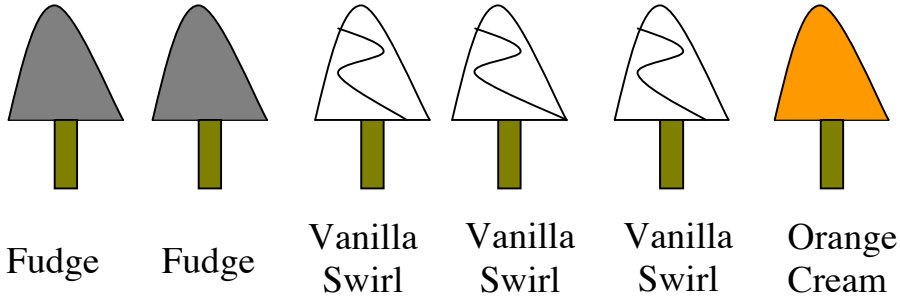
Additional Support Strategies: Consider using unifix cubes to show fraction multiplication. These can be helpful because you can break them apart and put them together. They provide a concrete, tactile representation of fractions of sets and linear measure.

RESOURCES:

- Everyday Mathematics, Unit 7
- [Progressions for the Common Core State Standards in Mathematics, \(draft\) 3-5 Number and Operations—Fractions](http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_nf_3_5_2011_08_12.pdf), 8/12/11 (http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_nf_3_5_2011_08_12.pdf)
- [Fishin' for Fractions](http://www.nsa.gov/academia/files/collected_learning/elementary/arithmetic/fish_for_fract.pdf) is a set of 3 lessons that addresses fractions as points on a number line (relating to length in inches) and engages students in comparing and ordering fractions. These lessons are available at (http://www.nsa.gov/academia/files/collected_learning/elementary/arithmetic/fish_for_fract.pdf)
- [LearnZillion](http://www.learnzillion.com) provides brief video tutorials that explain fraction equivalency and algorithms for fraction operations.
- [Khan Academy](http://www.khanacademy.org/) (<http://www.khanacademy.org/>) provides brief video tutorials that explain fraction equivalency and algorithms for fraction operations.
- *Elementary and Middle School Mathematics: Teaching Developmentally*. Van De Walle, John. New York: Pearson, 2004.
- Counters, rulers, fraction and whole number lines

Penguin Ice Cream Company

The Penguin Ice Cream Company sells assorted packages of ice cream.
The popular Penguin Bar is sold in packages of 6.



1. What fraction of the package is Fudge Bars? _____
What fraction of the package is Vanilla Swirl Bars? _____
What fraction of the package is Orange Cream Bars? _____
2. Most Penguin Ice Cream Sandwiches are in packages of 10. The package contains 4 chocolate, 4 vanilla, and 2 neapolitan. What fraction of the package is each?
_____ chocolate
_____ vanilla
_____ neapolitan

Melissa and her friends ate $\frac{2}{5}$ of a package after school, her brother Chad ate $\frac{1}{10}$ after soccer practice. What fraction of the package is left? _____
Show how you figured it out: _____

3. Penguin Real Fruit Bars are packed as follows: 3 lemon, 3 lime, 4 strawberry and 2 coconut.

What fraction of the package is each?

_____ lemon

_____ lime

_____ strawberry

_____ coconut

Show how you figured it out.

4. The economy pack of Penguin Ice Cream Sandwiches is sold in packages of 24. They are packed as follows: $\frac{1}{2}$ chocolate, $\frac{1}{4}$ vanilla, $\frac{1}{6}$ strawberry and the rest are neapolitan.

How many sandwiches of each flavor are in the package?

_____ chocolate

_____ vanilla

_____ strawberry

_____ neapolitan

Show how you figured it out.

Lesson Name/Overview	
Many Faces of Fractions	
Lesson Objective	
Students will show multiple representations of fractions using number lines, area models, equivalencies, and counter models.	
CCLS for Mathematical Content	CCLS for Mathematical Practice
<p><u>Number and Operations Fractions</u> 4.NF.1 Extend understanding of fraction equivalence and ordering.</p> <ol style="list-style-type: none"> 1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 	<p>MP.3: Construct viable arguments and critique the reasoning of others.</p> <p>MP.4: Model with mathematics.</p>
Essential Questions/Understandings	
<ul style="list-style-type: none"> ✓ How can we use number lines, area models, and counter models to show fractions? ✓ What does fraction equivalency mean? ✓ How can we use an algorithm to create equivalency? ✓ How can we prove to our classmates that our answers are reasonable and correct? 	
Materials	Vocabulary
<ul style="list-style-type: none"> ▪ Index cards ▪ Chart paper ▪ Rulers ▪ Counters ▪ Markers and crayons ▪ Grid paper ▪ Tape 	<ul style="list-style-type: none"> ▪ Algorithm ▪ Equivalent fractions ▪ Area model
Task	
<ul style="list-style-type: none"> • Students will be given 5 different fractions such as: $5/9$, $2/5$, $11/8$, $6/11$, and $3/7$. • Students will be responsible for creating an index card showing a representation of each fraction using a different type of model. Thus, for each fraction, students will create one model. • For example, a student can show $2/5$ on a number line. • Then the student might show $3/7$ using an area model. This model might also show equivalency for $3/7$. • Also, the student might show equivalency for $6/11$ using an algorithm. • Finally, they could show $5/9$ using an area model. 	

- Since they must create a different model for each fraction, by the end of the lesson they will make four cards where each will show a number line, area model, algorithm, and counter model/picture.
- Students will meet with a partner to compare and explain their answers.
- As a partnership, they will go around the room taping their fraction representation to the appropriate chart. (Teacher will have charts labeled with each fraction.)
- They will walk around the room looking at the fraction representations that are displayed, noting patterns, and thinking of questions.
- They can write a summary of what they did, as well as include their partner discussions and how they explained their answers to each other.

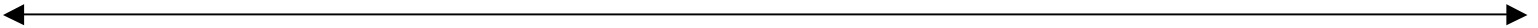
Possible Misconceptions	Additional Support Strategies
<ul style="list-style-type: none"> • Students may be confused about how to represent the fractions and how many models they need to make. 	<ul style="list-style-type: none"> • Teacher can show an example of representations for different fractions that the students will not be working with.
Possible Questions to Assess or Advance Thinking	
<ul style="list-style-type: none"> • What patterns did you notice in the representations? • How do models help us understand fractions? 	
Set-up/Explore/Share	
<ul style="list-style-type: none"> • Teacher must set up the supplies and chart paper for students to work with around the room. • Students should have a chance to work with different representations and tools such as rulers and counters. • Students should come together to discuss patterns they saw and questions they have. 	

Name: _____

Date: _____

Cuisenaire Rods Worksheet: Number Lines

Directions: Pick the purple rod as the **whole**. If this rod is the **whole**, then what part of the **whole** would each rod represent? Only use the red, light green, and white rods as fractional parts. Label each piece with the color and **fractional part** it represents. Show this on the number line below.



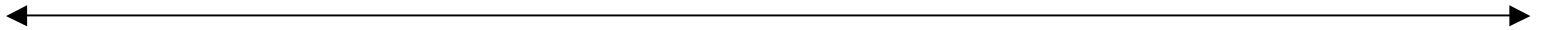
Think about what fractional part the pieces greater than the **whole** would represent.

Name: _____

Date: _____

Cuisenaire Rods Worksheet: Number Lines

Directions: Pick a rod to equal the **whole**. If this rod is the **whole**, then what part of the **whole** would each rod represent? Label each piece with the color and **fractional part** it represents. Show this on the number line below.



Think about what fractional part the pieces greater than the **whole** would represent

Name: _____

Date: _____

FORMATIVE ASSESSMENT

Comparing Fractions

Jaida and Anaya go to the pie shop. They each get cherry pies that are the same size. Jaida eats $\frac{2}{3}$ of her pie. Anaya eats $\frac{3}{4}$ of her pie. Jaida says they ate the same amount of pie, but Anaya says she ate more. Who is correct?

Use a model and words to explain your answer.

My answer: _____

Model	Words

Are you finished? Make up your own word problem comparing fractions with unlike denominators and challenge your partner to solve it.

Name: _____

Date: _____

Area Models and Equivalent Fractions

1. Use a rectangular area model to find three equivalent fractions for $\frac{1}{5}$.

2. Which of the following fractions is greater, $\frac{2}{3}$ or $\frac{5}{6}$? Use an area model to justify your answer.

3. Generate a fraction equivalent to $\frac{5}{6}$ and explain why it is equivalent.

Name: _____

Date: _____

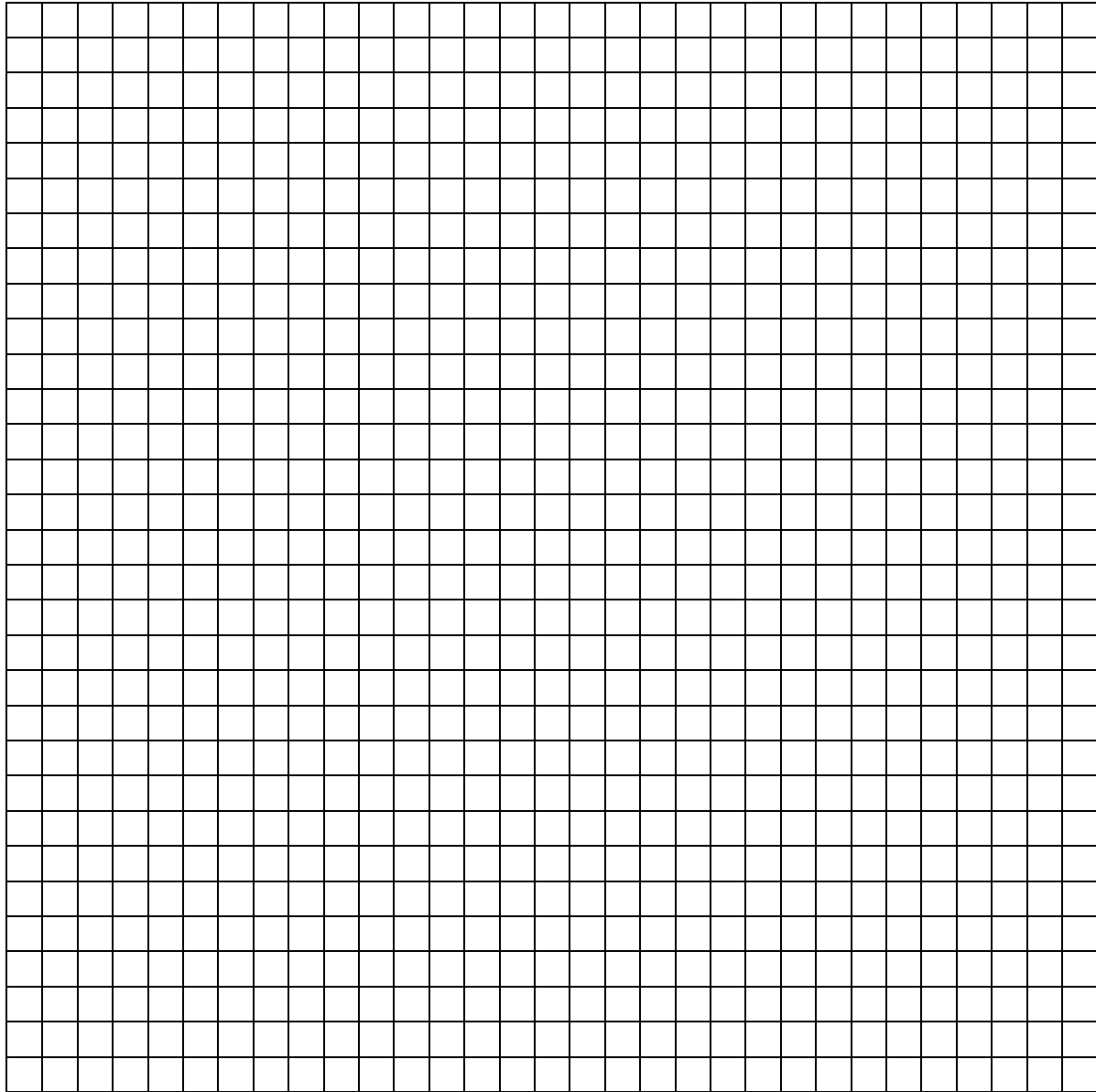
INSTRUCTIONAL TASK

A Trip to the Candy Store

Adara and Elizabeth go to the candy store, and each buys a giant candy bar. The candy bars are the same size. Adara's candy bar is cut into 27 slices and she eats 21 of them. Elizabeth's candy bar is cut into 9 slices and she eats 7 of them.

Who ate more? _____

Use an area model and words to explain your answer. *You may use the attached grid to assist as you draw your area model.*



Lesson Name/Overview	
Fraction Number Line Multiplication In this lesson students will connect their understandings of fractions as numbers on a number line to their understanding of how multiplication of whole numbers can be represented as “hops” on a number line.	
Lesson Objective	
Students will use number lines to multiply fractions by whole numbers.	
CCLS for Mathematical Content	CCLS for Mathematical Practice
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. 4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. <ol style="list-style-type: none"> Understand a fraction a/b as a multiple of $1/b$. <i>For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.</i> 	MP.1: Make sense of problems and persevere in solving them. MP.7: Look for and make use of structure. MP.8: Look for and express regularity in repeated reasoning.
Essential Questions/Understandings	
<ul style="list-style-type: none"> ✓ How can you use a number line to show multiplication? ✓ How is multiplication of whole numbers related to multiplication of fractions by whole numbers? 	
Materials	Vocabulary
<ul style="list-style-type: none"> ▪ Fraction number line multiplication activity packet 	<ul style="list-style-type: none"> ▪ equation ▪ numerator ▪ denominator ▪ number line ▪ fraction ▪ whole number ▪ multiplication
Task	
<ul style="list-style-type: none"> • See attached “Fraction Number Lines” for task 	
Possible Misconceptions	Additional Support Strategies
<ul style="list-style-type: none"> • Students may not use the number lines to solve problems. • Students may not know how to write a multiplication equation to represent their thinking. • Students may multiply the numerator and denominator by the whole number factor. 	<ul style="list-style-type: none"> • Model skip counting on the number lines. • Support students by modeling the language of fractions (pointing to and naming the fractional part that is referenced). • Provide other representations for students (e.g. area models of fractions).

Possible Questions to Assess or Advance Thinking

- How do the numerator and denominator of a fraction compare to the product when the fraction is multiplied by a whole number? (Point to the parts of a fraction and the whole number factor to support students in answering the question.)
- How can you use the number line to show multiplication? (Point to whole number multiplication.)
- How can you use the number line in a similar way to show the multiplication of a fraction by a whole number?
- What patterns did you notice in the multiplication equations you wrote?

Set-up/Explore/Share

- Students will be given fraction number line multiplication activity packet.
- Based on previous assessments, students will be grouped with a partner with similar understanding of fractions.
- Students work through the packet with their partners.
- As students work on the task, circulate and note where students are having difficulty.
- Teacher will facilitate whole class and group discussions around the essential questions.
- Students will be asked to explain their thinking and give examples of their work.

Name: _____

Date: _____

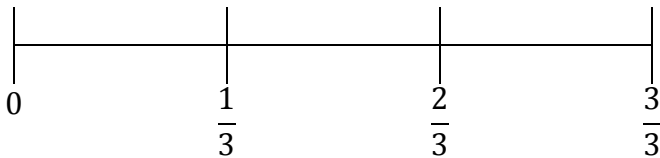
Fraction Number Lines

How much is 3 groups of 1?



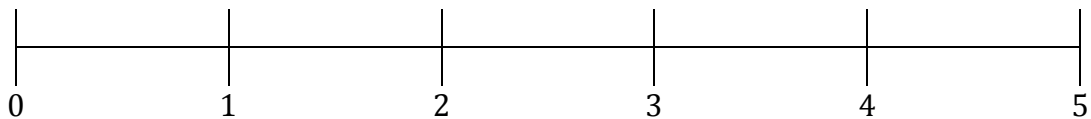
Write a multiplication equation to represent your thinking.

How much is 3 groups of $\frac{1}{3}$?



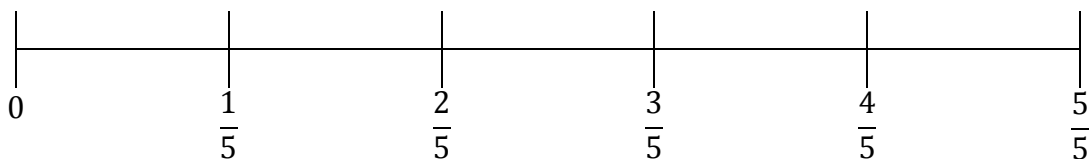
Write a multiplication equation to represent your thinking.

How much is 4 groups of 1?



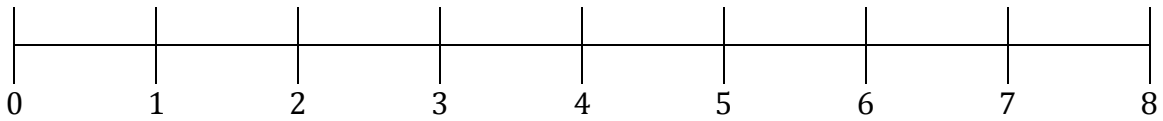
Write a multiplication equation to represent your thinking.

How much is 4 groups of $\frac{1}{5}$?



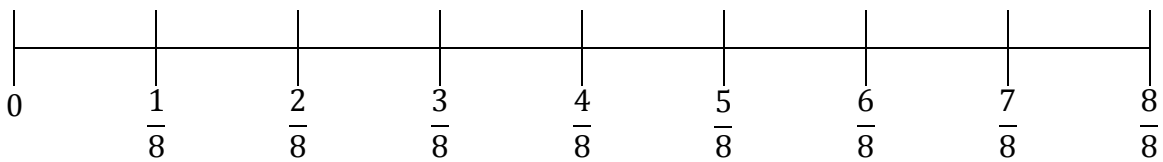
Write a multiplication equation to represent your thinking.

How much is 3 groups of 2?



Write a multiplication equation to represent your thinking.

How much is 3 groups of $\frac{2}{8}$?



Write a multiplication equation to represent your thinking.

How are multiplication equations with whole numbers related to multiplication equations with fractions? What patterns do you notice?

Name: _____

Date: _____

INSTRUCTIONAL TASK

Jetta and Caitlyn's Brownie Task

There are 3 brownies on a plate. Jetta, Caitlyn, and Samantha each ate $\frac{3}{4}$ of a brownie. How many brownies did they eat all together? You may use purple, red, white, and green Cuisenaire Rods to solve.

Use a model, number line, and words to explain your answer.

My answer: _____

Model	
Number Line	
Words	

Name: _____

Date: _____

FORMATIVE ASSESSMENT

Kelley's Pizza Shop

Kelley's pizza shop sells slices that are $\frac{1}{4}$ of a pie. Here are their sales for the week. Fill in the missing information in the table below.

Day of Week	Slices of Pizza Sold	Amount of Pies Sold
Monday	2	
Tuesday	3	
Wednesday	6	
Thursday	8	
Friday	7	
Saturday		3

Use this space for your work.

Explain how you determined how many pizza pies were sold each day.
